

Feynman Lectures on Physics Vol. II offers the most general to the standard wave equation:

$$\partial^2 \phi(x) = 0$$

Given a spacetime position  $x = (t, x_1, x_2, x_3, \dots)$

Most general solution:

$$\phi(x) = f(k \cdot x)$$

(Linear combinations of different choices of  $f$  and  $k$  are possible,  $f$  could be any (well-behaved) function. I focussed on monochromatic light.)

Using speed of light  $c = 1$ , and light-like vector  $k^2 = 0$ .

This general case means that we have wavefunction travelling forward along  $k$ , carrying a electromagnetic pattern/signal of arbitrary shape, (arbitrary shape orthogonal to  $k$ ).

## 0.1 General wave function In any dimension

Define:

$k = e_0 + e_1$  (Light-like vector:  $k^2 = 0$ , we just choose  $e_1$  as the direction of the EM wave)

Assume we have functions:  $f_2(\tau), f_3(\tau), f_4(\tau), \dots$

$$A(x) = \sum_{i=2}^N f_i(k \cdot x) e_i$$

$A$  is our  $N$ -dimensional EM vector potential.

Then  $F = \partial \wedge A$

$$F(x) = \sum_{i=2}^N f'_i(k \cdot x) k \wedge e_i$$

$$F(x) = k \sum_{i=2}^N f'_i(k \cdot x) e_i$$

Hence this bivector is incident to  $k$ .

The spacetime split shows how the electric bivector is incident to time  $e_0$ , and the magnetic bivector only features space dimensions:

$$F(x) = E(x) + B(x)$$

$$E(x) = \sum_{i=2}^N f'_i(k \cdot x) e_0 e_i$$

$$B(x) = \sum_{i=2}^N f'_i(k \cdot x) e_1 e_i$$

This also means that both the electric bivector and magnetic bivector lie incident to  $k$ . (And from this we would ordinarily deduce that the magnetic field VECTOR is perpendicular to the electric vector, so I think this tracks.)

We can now work with any space & time dimensions.

## 0.2 Example: Linear polarized Light

Example wave solution:

$$f_2(\tau) = C \cos(\tau)$$

$$A(x) = f_2(k \cdot x) e_2$$

$$\partial^2 A = -C \cos(x) k^2 e_2 = 0$$

(Because direction of propagation is null vector:  $k^2 = 0$ )

Now the Electromagnetic Field  $F$ :

$$F(x) = \partial \wedge A(x) = -\sin(k \cdot x) k e_2$$

Perform a spacetime split into Electric & magnetic components:

$$F(x) = E(x) + B(x)$$

$$E(x) = C \sin(k \cdot x) e_0 e_2$$

$$B(x) = C \sin(k \cdot x) e_1 e_2$$

Thus we can deduce  $E$  and  $B$  from our vector-potential  $A$ .

## 0.3 Circular Polarized Light

Now we use a rotating potential:

$$A(x) = C \exp(k \cdot x e_2 e_3) e_2 \exp(-k \cdot x e_2 e_3)$$

Again:  $k = e_0 + e_1$

And scalar constant  $C$

From this we get:

$$F(x) = \partial \wedge A(x) = k \exp(k \cdot x e_2 e_3) C \exp(-k \cdot x e_2 e_3)$$

spacetime split:

$$E(x) = C(\cos(k \cdot x) e_2 + \sin(k \cdot x) e_3) e_0$$

$$B(x) = c(\cos(k \cdot x) e_2 + \sin(k \cdot x) e_3) e_1$$

In  $N$ -dimensional space, polarization of a given light-wave simply has  $N - 1$  DOFs. However the polarization will still only be an ellipse, which can now be tilted in different ways.

## 0.4 Infinite Dimensional Case

In  $\infty$ -dimensional space, we have  $\infty$ -dim (Co-dimension 1) directions, in which the EM wave could be polarized. However, the rotating EM-field still only fol-

lows an ellipse, of a given frequency. (Assuming its monochromatic light.)

## 0.5 Multiple Time Dimensions

If we have 2 time dimensions, the wave vector  $k$  would still have a single direction. The most interesting part, is that light would have 2 frequencies:  $k = (\omega_0, \omega_1, k_1, k_2, k_3)$ ,  $k$  is still a light-like vector:  $k^2 = 0$ . and light would now not only travel along different spatial dimensions, but also along different time directions.

Lets say we have light:  $F(x) = C \exp(k_l \cdot t)$

And we have an observer, travelling along a different, more time-like vector:  $k_o$  (Now  $k_o < 0$ .) Then if the light hits the observer, the observer perceives the projection of the lights wavefunction along his/her spacetime trajectory. For the frequency we simply have:  $k_l \cdot k_o$ .

Assuming the observer has a low velocity (much lower than the speed of light), the apparent frequency of the light is simply the projection of the lights wave number  $k$  onto the observers "time". Thus if the lights time-component is almost orthogonal to our time, it will be very redshifted, if visible at all.

One interesting thing about multiple time dimensions, is that the electric field has bivector components, that lie in the time-plane.

I haven't looked into that yet.

\*We could also consider a field  $f_\perp(k \cdot x)$  along the  $e_0 - e_1$  direction, but this can be ignored

due to the Lorenz gauge:

$$\partial \cdot f_\perp(k \cdot x)(e_0 - e_1) = f'_\perp(k \cdot x)k \cdot (e_0 - e_1) = -2f'_\perp(k \cdot x) = 0$$

Thus:  $f'_\perp(k \cdot x) = 0$ .